Workplace Education Manitoba

NUMERACY:

The Basics Workbook

Set U: Geometry 3 Volume

Companion Workbook to Numeracy: The Basics Video Series

Watch the video for this workbook at
www.wem.mb.ca/learning_on_demand.aspx

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Workplace Education Manitoba

INTRODUCTION

What is Numeracy: The Basics Workbook?

This workbook is intended to accompany Workplace Education Manitoba's (WEM) Numeracy: The Basics Video Series, a set of 50 videos that explain essential numeracy concepts.

The refresher videos cover 25 critical numeracy topics, each broken into concept and practice.

The video series and accompanying downloadable workbooks can be found on the WEM website at http://www.wem.mb.ca/learning_on_demand.aspx

These Numeracy: The Basics workbooks provide an opportunity for additional skill-building practice.

Numeracy: The Basics topics are:

- Order of Operations 1
- Order of Operations 2
- Adding & Subtracting Fractions 1
- Adding & Subtracting Fractions 2
- Multiplying & Dividing Fractions
- Mixed & Improper Fractions
- Operations with Mixed Fractions 1
- Operations with Mixed Fractions 2
- Operations with Mixed Fractions 3
- Adding & Subtracting Decimals
- Multiplying Decimals
- Dividing Decimals
- Order of Operations & Decimals
- Decimals, Fractions & Percent 1
- Decimals, Fractions & Percent 2
- Imperial Conversions
- Metric Conversions
- Metric and Imperial Conversions
- Geometry 1 – Perimeter
- Geometry 2 – Area
- Geometry 3- Volume
- Solving Equations 1
- Solving Equations 2
- Ratio & Proportion
- Averages

GEOMETRY 3 VOLUME

This workbook contains five skill-building practice sections. Solutions can be found at the end of the workbook.

Practice Section A

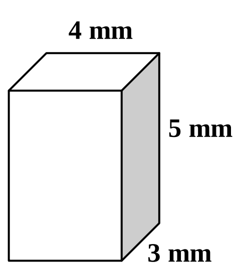
Solve the following.

1. Calculate the volume of a cube that has a side length of 4 cm.

2. What is the volume of a figure that has a length of 5 ft, a width of 2 ft, and is 7 ft high?

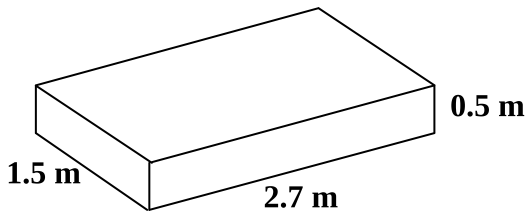
3. Find the volume of a cube that is 0.5 yd on each side.

4. Calculate the volume of the figure below.



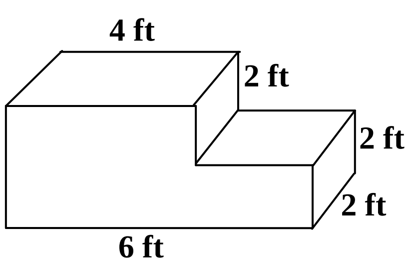
5. A cube has sides equal to 7 m. Find its volume.

6. Determine the volume of the object in the diagram below.

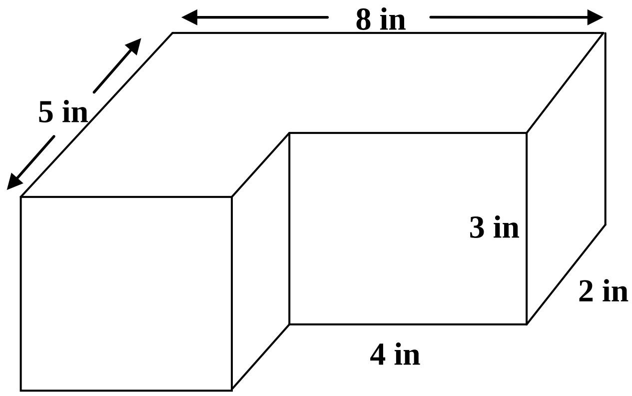


7. A box with a square base has a height of 7 m. If each side of the base is 2 m in length, calculate the volume of the box.

8. Find the volume of the figure below.



9. Find the volume of the figure below.



10. How many small boxes can fit into a larger box that is 2 times (double) bigger in every dimension?

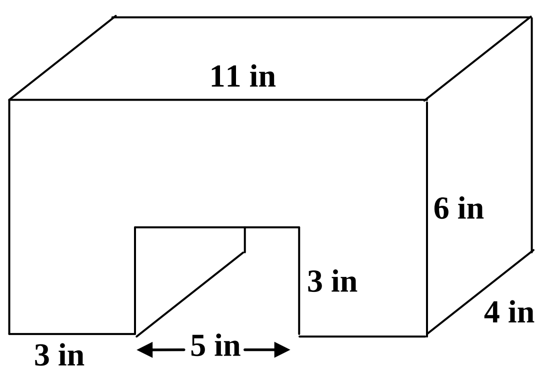
Practice Section B

Solve the following.

1. A box used for shipping fragile goods has a volume of 8 ft^3. How many closely packed smaller boxes, each with a volume of 0.625 ft^3 will fit into the larger box?

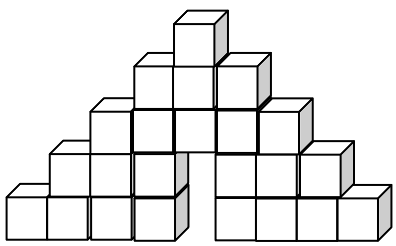
2. A tall box has a volume of 68 cm^3, a length of 8.5 cm, and a width of 4 cm. Find the measure of the height.

3. Calculate the volume of the figure below.



4. How many boxes, each with a volume of 60 mm^3, can fit in a cubed-shape box with a side length of 10 mm?

5. The figure below is made from blocks that are 2 cm long, 1 cm wide, and 1 cm high. Find the total volume of all of the blocks.

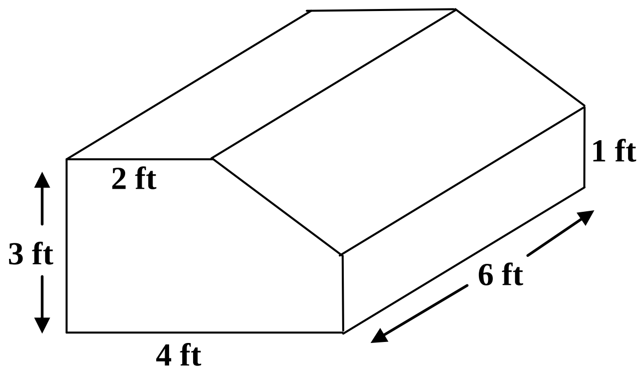


Practice Section C

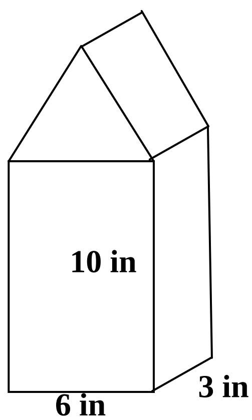
Solve the following. Round each answer to two decimal places, if rounding is necessary. Note that diagrams are not drawn to scale.

1. The volume of a rectangular solid is 675 cm^3. If the height is 25 cm and the base is a rectangle whose length is 3 times the width, what are the dimensions of the base?

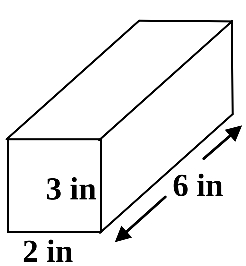
2. Determine the volume of the figure below.



3. The triangular peak in the figure below is 20% of the volume of the entire container. What is the volume of the figure below?



4. A 1 cm thick Styrofoam sheet is to be placed around every small box (on all sides). These small boxes are packaged into one larger box that is a 4 ft cube. The diagram below is of one of the smaller boxes. How many smaller boxes can the larger box hold?



5. There are 16 holes that must be filled with concrete. Each hole is an 8 inch square that is 2 meters deep. Concrete costs $84.25 per cubic yard. How much will it cost to fill all the holes with concrete?

Practice Section D

In this section, solutions for the practice questions contain commonly-made errors. For each question, circle the error(s) and give a correct solution.

1. Sarah thinks that there is only one cube whose volume is equal to its surface area. Sarah's explanation is below.

If the volume and the surface area of a cube are equal, and there are 6 sides to a cube, then:

$$V_{cube} = A_{cube}$$
$$\cancel{l} \times \cancel{w} \times h = 6 \times \cancel{l} \times \cancel{w}$$
$$h = 6$$

Therefore, the height of the cube must be 6. Given the definition of a 'cube,' the length and the width must also be 6. Only a cube with a height of 6 will have its volume equal to its surface area.

Is Sarah correct? Explain.

Practice Section E
Challenge Question. If you can do this one, then you get an A^+. ☺

Very fragile light bulbs are packaged into rectangular boxes that are 6 inches long, 3 inches wide, and 2 inches tall. The light bulb takes up 45% of the volume of the empty box. There is a 2 mm Styrofoam liner inside the box. What is the empty volume inside the box if it contains the light bulb and the liner?

SOLUTIONS

Set U

Geometry 3 Volume

GEOMETRY 3 VOLUME

Practice Section A

1. Solution:

$V = l \times w \times h$

$= 4 \times 4 \times 4$

$= 64\, cm^3$

2. Solution:

$V = l \times w \times h$

$= 5 \times 2 \times 7$

$= 70\, ft^3$

3. Solution:

$V = l \times w \times h$

$= 0.5 \times 0.5 \times 0.5$

$= 0.125\, yd^3$

4. Solution:

$V = l \times w \times h$

$= 4 \times 3 \times 5$

$= 60\, mm^3$

5. Solution:

$V = l \times w \times h$

$= 7 \times 7 \times 7$

$= 343\, m^3$

6. Solution:

$V = l \times w \times h$

$= 2.7 \times 1.5 \times 0.5$

$= 2.025\, m^3$

7. Solution:

$$V = l \times w \times h$$
$$= 2 \times 2 \times 7$$
$$= 28\,m^3$$

8. Solution:

This figure can be divided into a two boxes. The larger box (on the left) is 4 ft long, 4 ft tall, and 2 ft wide. The smaller box will be a 2 ft cube.

The total volume of the figure is:

$$V_{total} = (l \times w \times h) + (l \times w \times h)$$
$$= (4 \times 2 \times 4) + (2 \times 2 \times 2)$$
$$= 32 + 8$$
$$= 40\,ft^3$$

9. Solution:

To find the volume of the figure, calculate the volume of the entire box (as if there is no 'chunk' taken out) and then subtract the volume of the 'chunk' from the total.

The volume of the entire box is:

$$V_{box-no\,chunk} = l \times w \times h$$
$$= 8 \times 5 \times 3$$
$$= 120\,in^3$$

The volume of the 'chunk' is:

$$V_{chunk} = l \times w \times h$$
$$= 4 \times 3 \times 3$$
$$= 36\,in^3$$

The volume of the figure is:

$$V_{figure} = V_{box-no\,chunk} - V_{chunk}$$
$$= 120 - 36$$
$$= 84\,in^3$$

10. Solution:

If the larger box is 2 times bigger in every dimension, and there are three dimensions, (length, width and height), then there will be 2^3 or $2 \times 2 \times 2$ or 8 small boxes that can fit into the larger box.

Practice Section B

1. Solution:

$$\frac{8\ \cancel{ft^3}}{1\,big\ box} \times \frac{1\,small\ box}{0.625\ \cancel{ft^3}} = 12.8\frac{small\ box}{1\,big\ box}$$

Since you cannot have 0.8 of a box, 12 small boxes will fit into the larger box.

2. Solution:

$$V = l \times w \times h$$
$$68 = 8.5 \times 4 \times h$$
$$68 = 34 \times h$$
$$h = \frac{68}{34}$$
$$h = 2\,cm$$

3. Solution:
The object can be divided into three sections. Once the volume of each section is calculated, the total volume can be found by adding all volumes together.

There are two sections that measure 3 by 4 by 6 and one section that measures 5 by 3 by 4.

$$V_{total} = 2(l \times w \times h) + (l \times w \times h)$$
$$= 2(4 \times 3 \times 6) + (5 \times 3 \times 4)$$
$$= 2(72) + (60)$$
$$= 144 + 60 = 204\,in^3$$

4. Solution:
A 10 mm cube has a volume of:

$$V = l \times w \times h$$
$$= 10 \times 10 \times 10$$
$$= 1000\,mm^3$$

Since each box is 60 mm^3,

$$\frac{1000\ \cancel{mm^3}}{60\ \cancel{mm^3}} = 16.\overline{6}$$

Since it is not possible to have $0.\overline{6}$ of a box, there would be room for 16 boxes.

5. Solution:
There are a total of $1+3+5+6+8=23$ blocks.

Each block has a volume of:

$$V = l \times w \times h$$
$$= 1 \times 1 \times 2$$
$$= 2\,cm^3$$

Multiply the volume of one box by the total number of blocks to find the total volume.

$$23\,\cancel{blocks} \times \frac{2\,cm^3}{1\,\cancel{block}} = 46\,cm^3$$

Practice Section C

1. Solution:

$$V = l \times w \times h$$
$$675 = l \times w \times 25$$
$$\frac{675}{25} = l \times w$$
$$27 = 3w \times w$$
$$27 = 3w^2$$
$$\frac{27}{3} = w^2$$
$$w^2 = 9$$
$$w = \sqrt{9}$$
$$w = 3\,cm$$

Therefore, the dimensions of the base are 3 cm wide and 9 cm long.

2. Solution:
It is easiest here to divide the figure into a rectangular box with dimensions 2 by 3 by 6 and a volume of:

$$V = l \times w \times h$$
$$= 6 \times 2 \times 3$$
$$= 36\,ft^3$$

Another box can be made that measures 2 by 1 by 6 and that has a volume of:

$$V = l \times w \times h$$
$$= 6 \times 2 \times 1 \quad .$$
$$= 12\,ft^3$$

The volume that remains is actually half (only if the top of the figure is 'sliced' along its diagonal) of a 2 by 2 square box with a height of 6 ft.

$$V = \frac{1}{2}(l \times w \times h)$$

Its volume is: $= \frac{1}{2}(2 \times 2 \times 6)$

$$= 12\,ft^3$$

The total volume is the sum of the three volumes.

$$V = 36 + 12 + 12$$
$$= 60\,ft^3$$

3. Solution:
The volume of the rectangular box measuring 6 by 3 by10 is 180 in^3.
If the triangular cap is 20% of the entire container, then 180 in^3 must be 80%.
If 80% is 180 in^3, then 40% is 90 in^3, and so 20% is 45 in^3.

Therefore, the volume is:

$$V = 20\% + 80\%$$
$$= 45 + 180$$
$$= 225\,in^3$$

4. Solution:
Each smaller box has a volume of 6 by 2 by 3 or 36 in^3.

A 1 cm thick piece of foam will make each of these dimensions $1\,\cancel{cm} \times \frac{1\,in}{2.54\,\cancel{cm}} = 0.3937\,in$ larger on each side.

Therefore, you need to add $2 \times 0.3937 = 0.7874\,in$ to each measure.

This makes the volume of a packed box:

$$V = (6+0.7874)(2+0.7874)(3+0.7874)$$
$$= 6.7874 \times 2.7874 \times 3.7874$$
$$= 71.6545 in^3$$

Next, convert 4 feet to inches.

$$= 4\cancel{ft} \times \frac{12 in}{1\cancel{ft}}$$
$$= 48 in$$

Now calculate the volume of the larger box.

$$V_{l\arg er\,box} = 48 \times 48 \times 48$$
$$= 110592 in^3$$

Using a unit conversion,

$$\# box = \frac{110592\cancel{in^3}}{1 l\arg e\,box} \times \frac{1 small\,box}{71.6545\cancel{in^3}}.$$
$$= 1543.41$$

Since we cannot have 0.41 of a box, there are 1543 small boxes that can fit into a larger 4 ft cube box.

5. Solution:

First convert 8 inches to yards.

$$8\cancel{in} \times \frac{1\ \cancel{foot}}{12\ \cancel{in}} \times \frac{1\,yd}{3\ \cancel{feet}} = 0.\overline{2}\,yd$$

Next convert 2 meters to yards.

$$2\cancel{m} \times \frac{100\cancel{cm}}{1\cancel{m}} \times \frac{1\cancel{in}}{2.54\cancel{cm}} \times \frac{1\cancel{ft}}{12\cancel{in}} \times \frac{1\,yd}{3\cancel{ft}} = 2.1872\,yd$$

The total volume of concrete needed is the volume of the hole multiplied by the total number of holes.

$$V_{concrete} = V_{hole} \times \#_{holes}$$
$$= (l \times w \times h) \times 16$$
$$= (0.\overline{2} \times 0.\overline{2} \times 2.1872) \times 16$$
$$= 1.7281\,yd^3$$

The cost of the concrete is:

$$Cost = V_{concrete} \times \frac{\$84.25}{1\,yd^3}$$

$$= 1.7281\,\cancel{yd^3} \times \frac{\$84.25}{1\,\cancel{yd^3}}$$

$$= \$145.59$$

Practice Section D

1. Solution:
Sarah is correct. Given the specification of a ‘cube,’ rather than just a ‘box,’ all sides must be equal. If the height must be 6, then all the sides must also be 6.

Practice Section E

Solution:
The volume of an individual box is 6 by 3 by 2 or 36 in^3 (with no liner).
Each dimension of the box will get 'double' the thickness of the Styrofoam.
Once the 2 mm is converted to inches, the dimensions of the inner box will be:

Length

$$6in - 4\,\cancel{mm} \times \frac{1\,\cancel{cm}}{10\,\cancel{mm}} \times \frac{1in}{2.54\,\cancel{cm}}$$
$$= 6in - 0.1574in$$
$$= 5.8425in$$

Width

$$3in - 4\,\cancel{mm} \times \frac{1\,\cancel{cm}}{10\,\cancel{mm}} \times \frac{1in}{2.54\,\cancel{cm}}$$
$$= 3in - 0.1574in$$
$$= 2.8425in$$

Height

$$2in - 4\,\cancel{mm} \times \frac{1\,\cancel{cm}}{10\,\cancel{mm}} \times \frac{1in}{2.54\,\cancel{cm}}$$
$$= 2in - 0.1574$$
$$= 1.8425in$$

This will give a total inner volume of:

$$V = 5.8425 \times 2.8425 \times 1.8425$$
$$= 30.5989in^3$$

The volume of the light is 45% of the volume with no liner.

$$V = 45\% \times 36$$
$$= 0.45 \times 36$$
$$= 16.2in^3$$

The empty space in the box (assuming the shape of the Styrofoam is not altered by the insertion of the light bulb) will be:

$$V = 30.5989 - 16.2$$
$$= 14.3989in^3$$