INTRODUCTION

Ratio/Proportion is one of the most important mathematical applications you will use as a trades worker. You can use ratio/proportion to calculate increases and decreases, figure out percentages, compare rates and calculate rafter, gable stud and collar tie lengths.

When you have completed this chapter you will be able to:

- ❑ Write and simplify ratios.
- ❑ Write proportions and calculate the unknown quantity.
- ❑ Use proportion to solve similar triangles.
- ❑ Calculate direct proportions.
- ❑ Calculate indirect proportions.
- ❑ Compare two rates.
- ❑ Solve applications using ratio/proportion.

RATIOS

A ratio compares two numbers or quantities. For example, you can write a ratio to compare the amount of water to cement to make concrete. The two numbers in a ratio are called terms.

You can write ratios several ways:

Remember:

Always read ratios as 1 to 2.

1 to 2 1:2 $\frac{1}{2}$

For example, the ratio $\frac{1 \text{ water}}{2 \text{ cement}}$ would read 1 part water to two parts cement.

SIMPLIFYING RATIOS

Ratios should always be expressed in lowest terms.

Example 1:

10 to 30 = $\frac{10}{30} = \frac{1}{3}$

Although ratios should be expressed in lowest terms, improper ratios are left as improper ratios.

Example 2:

30 to 10 = $\frac{30}{10} = \frac{3}{1}$

Write whole number ratios as improper ratios.

Example 3:

3 to 1 = $\frac{3}{1}$

Hint:

Do not change the order of the terms in a ratio.

Write the following ratios in their lowest terms.

1) 6/3 2) 5/20 3) 9/27 4) 36/4

5) 48:12 6) 15:3 7) 8:2 8) 10:5

9) 27 to 9 10) 72 to 12 11) 10 to 150 12) 150 to 25

PROPORTION

A proportion compares two ratios. Proportions are usually written as a pair of equivalent fractions.

Example 1:

$\frac{1}{2} = \frac{5}{10}$ 1:2 :: 5:10 1 is to 2 as 5 is to 10

Always read a proportion as two equal ratios connected by the word 'as'.

Example 2:

1 is to 2 as 5 is to 10

The two cross products are equal in a proportion. Cross multiply to find the cross products.

Example 3:

$\frac{1}{2} = \frac{5}{10}$ $2 \times 5 = 1 \times 10$

$10 = 10$

There are two types of proportions: direct proportion and indirect proportion. The following table compares the difference between direct and indirect proportions:

Direct Proportion		Indirect Proportion	
$\frac{A}{B} = \frac{C}{D}$	$\frac{\text{increase}}{\text{decrease}} = \frac{\text{increase}}{\text{decrease}}$	$\frac{A}{B} = \frac{D}{C}$	$\frac{\text{increase}}{\text{decrease}} = \frac{\text{decrease}}{\text{increase}}$

Both direct proportion and indirect proportion are explained in this chapter.

DIRECT PROPORTION

A direct proportion compares two like quantities or two ratios that are directly proportional and equal to one another. This means an increase in one quantity leads to a proportional increase in the other quantity.

$$\frac{A}{B} = \frac{C}{D} \qquad \frac{\text{increase}}{\text{decrease}} = \frac{\text{increase}}{\text{decrease}}$$

Hint:

In a direct proportion, always make sure that whatever unit you place on the top in the first ratio is placed on the top in the second ratio.

SOLVING A DIRECT PROPORTION

Step 1: Set up a proportion.

Step 2: Substitute known values into the proportion and cross multiply.

Step 3: Divide to get the unknown value by itself.

Step 4: Check your answer by cross-multiplying. The answer on both sides of the equal sign should be equal.

Example:

If one hammer costs \$32.00, what would the cost of ten hammers be?

$\frac{A}{B} = \frac{C}{D}$ Set up the proportion.

$\frac{1 \text{ hammer}}{\$32.00} = \frac{10}{\chi}$ Substitute known values into the proportion and cross-multiply.

$\$32.00 \times 10 = 1 \times \chi$

$\frac{\$320.00}{1} = \chi$ Divide to get the unknown value by itself.

$\$320.00 = \chi$

$\$32.00 \times 10 = 1 \times \$320.00$ Check your answer by cross-multiplying.

$\$320.00 = \$320.00$

Ten hammers will cost \$320.00.

Calculate the missing quantity in the following direct proportions.

1) $\frac{1}{2} = \frac{\chi}{8}$ 2) $\frac{1}{3} = \frac{4}{\chi}$ 3) $\frac{5}{\chi} = \frac{10}{16}$ 4) $\frac{5}{15} = \frac{\chi}{60}$

5) $\frac{1}{5} = \frac{1.2}{\chi}$ 6) $\frac{\chi}{75} = \frac{30}{25}$ 7) $\frac{4}{7} = \frac{\chi}{56}$ 8) $\frac{15}{10} = \frac{\chi}{150}$

9) $\frac{6}{\chi} = \frac{2}{3}$ 10) $\frac{\chi}{20} = \frac{4}{5}$ 11) $\frac{1.5}{25} = \frac{\chi}{1012}$ 12) $\frac{5}{250} = \frac{25}{\chi}$

13) $\frac{\chi}{42} = \frac{2}{3}$ 14) $\frac{4}{9} = \frac{4.8}{\chi}$ 15) $\frac{7}{9} = \frac{\chi}{36}$ 16) $\frac{42}{3} = \frac{84}{\chi}$

Solving Similar Triangles

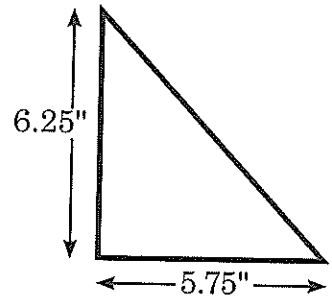
Triangles are similar if their matching angles are equal and the ratio of their corresponding sides is in proportion. Use proportion to calculate the length of the missing side in the similar triangles.

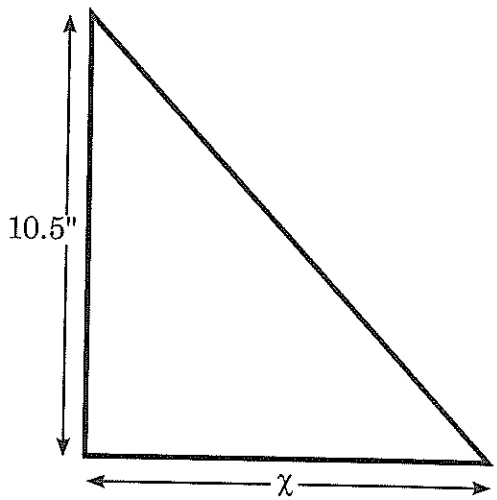
Step 1:	Draw a diagram and label it.
Step 2:	Set up a proportion with known values.
Step 3:	Cross-multiply.
Step 4:	Divide to get the χ alone.
Step 5:	Check your answer. Cross-multiply. Answers on both sides of the equal sign should be equal.

Hint:

If the problem compares two things and you can locate three values, you can write and solve a proportion.

Example:
The two triangles below are similar triangles. Calculate the length of the unknown side using a proportion.





$\frac{6.25}{5.75} = \frac{10.5}{\chi}$ Set up a proportion.

$6.25 \times \chi = 10.5 \times 5.75$ Cross-multiply.

$\chi = \frac{60.375}{6.25} = 9.66"$ Divide to get the χ alone.

$6.25 \times 9.66 = 10.5 \times 5.75$ Check your answer.

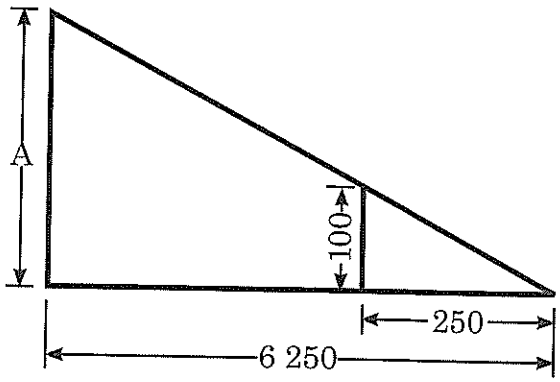
$60.375 = 60.375$ Cross-multiply.

The answers should be equal.

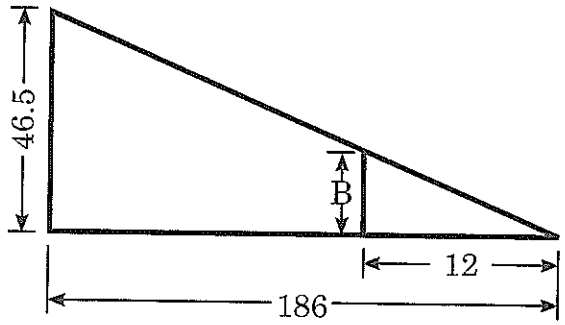
Calculate the missing length. All measurements are in millimetres. Round off answers to two decimal places.

1)

A =

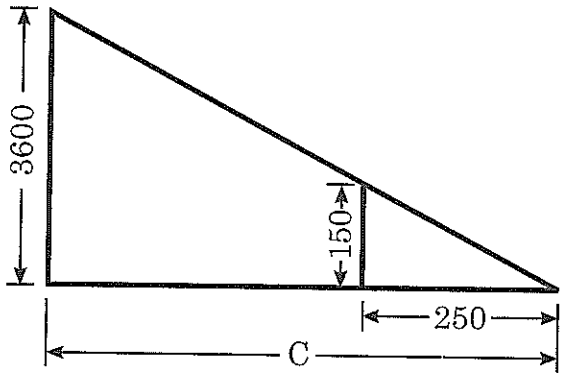


2)



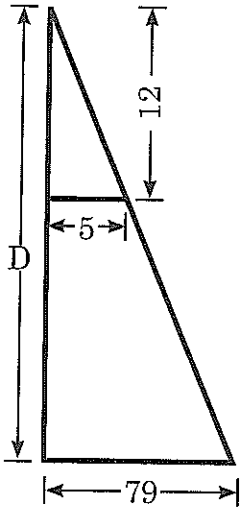
B =

3)



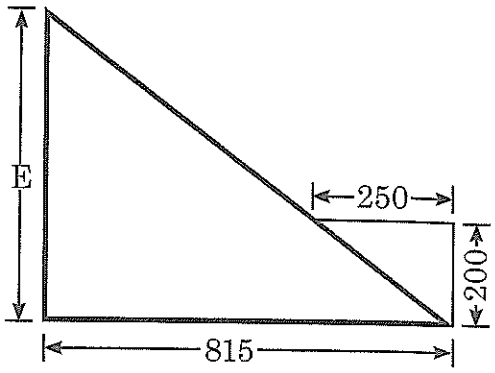
C =

4)



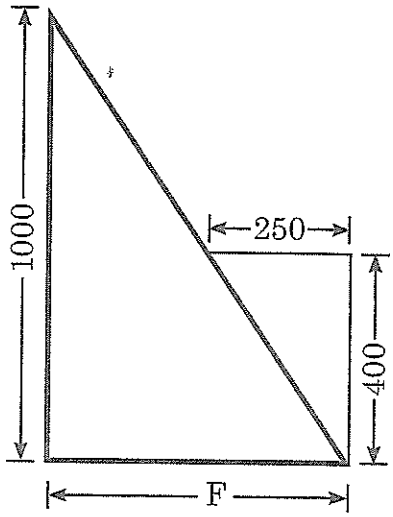
D =

5)



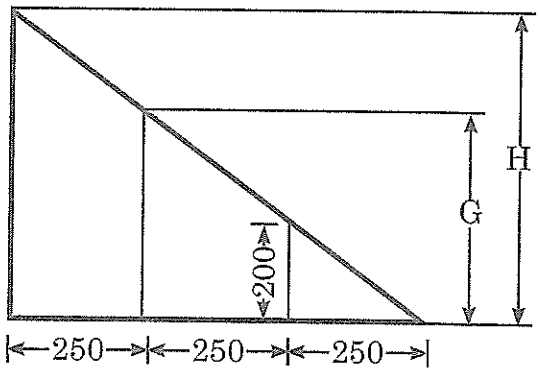
E =

6)



F =

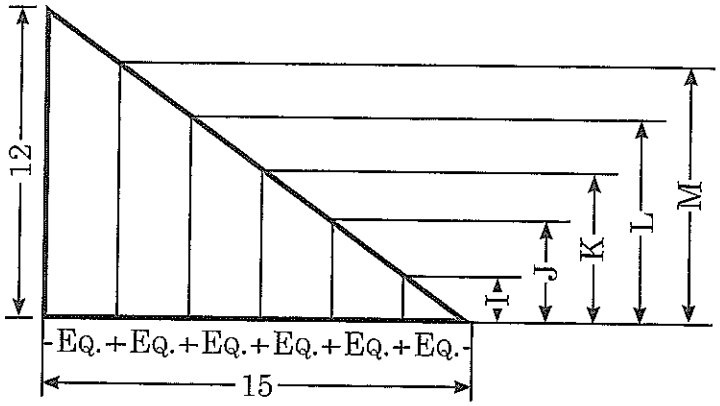
7)



G =

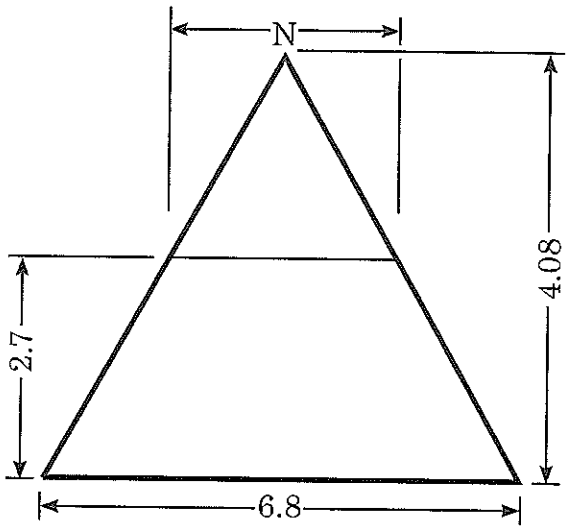
H =

8)



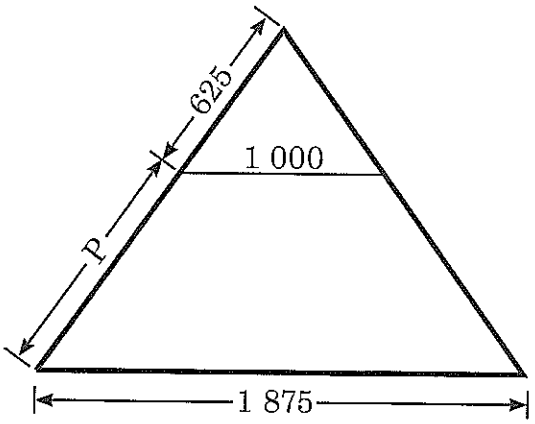
I =
J =
K =
L =
M =

9)



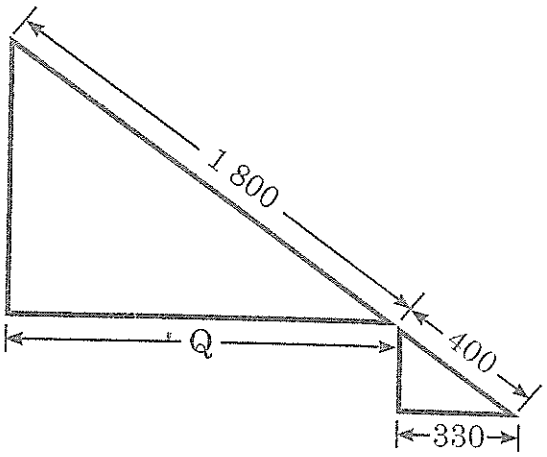
N =

10)



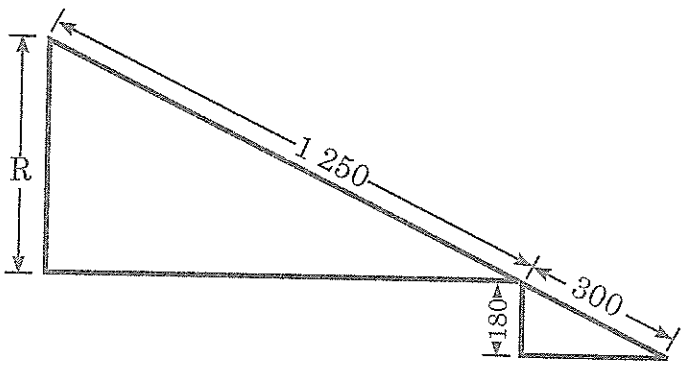
P =

11)



Q =

12)



R =

INDIRECT PROPORTION

An indirect proportion is a comparison between two quantities or ratios that are inversely proportional. This means an increase in one quantity leads to a decrease in the other quantity or when a decrease in one leads to an increase in the other. When you write an inverse proportion, either the first or the second ratio is inverted (flipped over).

$$\frac{A}{B} = \frac{D}{C} \qquad \frac{\text{increase}}{\text{decrease}} = \frac{\text{decrease}}{\text{increase}}$$

Solving a Indirect Proportion

You solve an indirect proportion the same way you solve a direct proportion. The only difference is in how you set up the proportion.

Step 1: Set up the proportion. Remember to invert either the first or second ratio.

Step 2: Substitute known values into the proportion and cross-multiply.

Step 3: Divide to get the unknown value by itself.

Step 4: Check your answer by cross-multiplying. The answer on both sides of the equal sign should be equal.

Example 1:

If it takes 3 carpenters 30 days to build one house, how many days would it take for 5 carpenters to build the same house?

If the number of carpenters is increased from 3 to 5, the number of days to complete the house will be decreased.

$\frac{A}{B} = \frac{D}{C}$ Set up the proportion. Remember to invert either the first or second ratio.

$\frac{3 \text{ carpenters}}{5 \text{ carpenters}} = \frac{\chi \text{ days}}{30 \text{ days}}$ Substitute known values into the proportion and cross-multiply.

$3 \times 30 = \chi \times 5$

$\frac{90}{5} = \chi$ Divide to get the unknown value by itself.

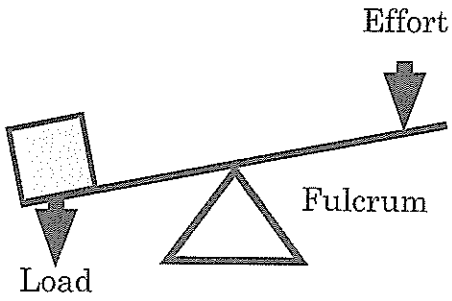
$18 = \chi$

$3 \times 30 = 18 \times 5$ Check your answer by cross-multiplying.

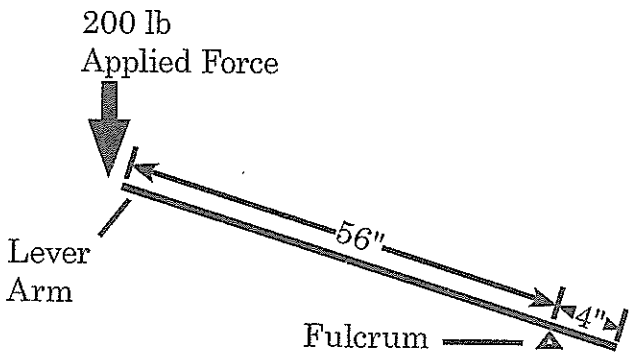
$90 = 90$

Example 2:

The diagram of the lever below was used to solve this indirect proportion. A lever is made up of a bar or rod that rests on a supporting point called a fulcrum. The fulcrum divides the lever into two parts called the effort arm and the resistance arm. Effort or force is applied to the effort arm which causes the resistance or load to move.



If a 200 pound force is applied to a 60 inch straight bar that is pivoted 4 inches from the end, what lift force is exerted? Hint: It is helpful to draw a diagram to solve a problem like this.



$\frac{A}{B} = \frac{D}{C}$ Set up the proportion.
Remember to invert either the first or second ratio.

$\frac{200 \text{ lbs applied force}}{\chi} = \frac{4 \text{ inch lever arm}}{56 \text{ inch lever arm}}$ Substitute known values into the proportion and cross-multiply.

$200 \times 56 = \chi \times 4$

$\frac{11\,200}{4} = \chi$ Divide to get the unknown value by itself.

$2\,800 \text{ lbs} = \chi$

$200 \times 56 = 28\,000 \times 4$ Check your answer by cross-multiplying.

$11\,200 = 11\,200$

The lift force is 2 800 lbs.

Calculate the missing value in the following indirect proportions.

1. If it takes 5 carpenters 80 days to build two houses, how many days would it take for 9 carpenters to build two houses?

2. If it takes 8 carpenters 26 days to complete a project, how many days would it take 11 carpenters to complete the same project?

3. If a 220 pound force is applied to a 64 inch straight bar that is pivoted 6 inches from the end, what lift force is exerted? Hint: It is helpful to draw a diagram to solve a problem like this.

4. If a 180 pound force is applied to a 60 inch straight bar that is pivoted 4.5 inches from the end, what lift force is exerted?

RATES

A rate is a ratio that compares two types of measurements. The denominator in a rate is usually 1. When you read a rate, you use the word 'per'.

Some common rates:

Kilometres per hour	100 km/hour means that in 1 hour you will travel 100 km
Millilitres per litre	Number of thousandths of a litre compared to one litre
Rpms per minute	Number of revolutions per minute.
Cost per unit	Cost per one unit

Using Rates to Compare

You can compare two rates when the same job is completed by two or more workers at different rates. The rate you are comparing is the length of time each worker takes. This type of comparison helps to estimate the costs of completing a job.

Step 1:	Write a ratio for each worker. Divide the top by the bottom.
Step 2:	Add the two answers.
Step 3:	Set up the proportion.
Step 4:	Substitute known values into the proportion and cross-multiply.
Step 5:	Divide to get the unknown value by itself.
Step 6:	Check your answer by cross-multiplying. The answer on both sides of the equal sign should be equal.

Example:

A journeyperson takes 2 hours to install a door including trim and hardware. An apprentice takes 3.5 hours to do the same job. If they are working together to install one door, how long will it take?

Journeyperson	Apprentice	Write a ratio for each worker. Divide the top by the bottom.
It takes a journeyperson 2 hours for 1 door.	It takes an apprentice 3.5 hours for 1 door.	
$\frac{1}{2} = 0.5$	$\frac{1}{3.5} = 0.2857$	
$0.5 + 0.2857 = 0.7857$ hours		Add the two answers.
$\frac{A}{B} = \frac{C}{D}$		Set up the proportion.
$\frac{\chi}{1 \text{ door}} = \frac{60 \text{ minutes}}{0.7857 \text{ hours}}$		Substitute the known values into the proportion and cross-multiply.
$\chi \times 0.7857 = 60 \times 1$		Divide to get the unknown value by itself.
$\chi = \frac{60}{0.7857}$		
$\chi = 76.365$ minutes		Convert to hours and minutes. Sixty minutes equals one hour.
$76.365 \div 60 = 1.27275$ hours		Convert 0.27275 to minutes. Multiply by 60.
$0.27275 \times 60 = 16.365$ minutes		

It will take the journeyperson and the apprentice 1 hour and 16 minutes to install one door if they are working together.

Calculate the missing value in the following proportion.

1) A journeyperson takes 3 hours to install two doors including trim and hardware. An apprentice takes 6.75 hours to install two doors. If they are working together to install two doors, how long will it take?

PRACTICE

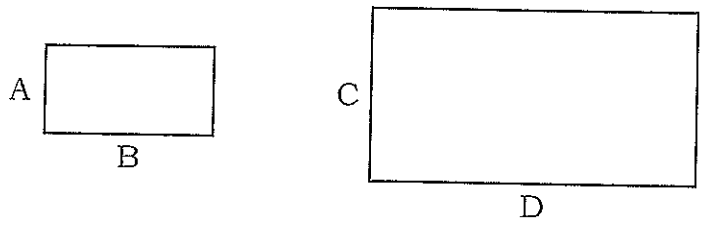
1) The concrete for a driveway is mixed at the ratio of 7 parts gravel to 2 parts cement. The foreman estimated 12 m^3 gravel is needed. Calculate the amount of cement needed. Round your answer to the nearest tenth of m^3.

2) The specified safety requirement for a sloped excavation is 0.75 horizontal to 1 vertical. Calculate the missing values in the table below.

	Horizontal	Vertical
a.	6'	
b.	8.5'	
c.		2 500 mm
d.		3 200 mm

3) If 3.6 kg of nails are used for each 90 m^2, how many kilos (kilograms) of nails are needed for 2 500 m^2?

4) The lengths of the two rectangles shown below are proportional to their widths. Complete the table below.



	A	B	C	D
a.		20 mm	54 mm	72 mm
b.	180 mm		480 mm	640 mm
c.	35 m	45 m		135 m
d.	22 m	38 m	48 m	

5) A proportion of 1:3:6 is used to mix cement, sand and gravel for concrete. How many cubic metres of each should be used to mix 86 m^3 of concrete?

Hint:

Indirect proportion.

6) If it takes 6 journeypersons 5 hours to complete a job, how long would it take 4 journeypersons to complete the same job?

Hint:

Indirect proportion.

7) If it takes 8 carpenters to complete a formwork project in 5 days, how many carpenters would it take to complete the same project in four days?

8) The ratio of the sides of a triangle is 3:4:5. The longest side is 8.75 cm. Calculate the lengths of the two other sides.

ANSWER KEY

RATIO AND PROPORTION

Page 86, **Simplifying Ratios**

1) $\frac{2}{1}$ 2) $\frac{1}{4}$ 3) $\frac{1}{3}$ 4) $\frac{9}{1}$

5) $\frac{4}{1}$ 6) $\frac{5}{1}$ 7) $\frac{4}{1}$ 8) $\frac{2}{1}$

9) $\frac{3}{1}$ 10) $\frac{6}{1}$ 11) $\frac{1}{15}$ 12) $\frac{6}{1}$

Page 89, **Solving a Direct Proportion**

1) $\chi = 4$ 2) $\chi = 12$ 3) $\chi = 8$

4) $\chi = 20$ 5) $\chi = 6$ 6) $\chi = 90$

7) $\chi = 32$ 8) $\chi = 225$ 9) $\chi = 9$

10) $\chi = 16$ 11) $\chi = 60.72$ 12) $\chi = 1\,250$

13) $\chi = 28$ 14) $\chi = 10.8$ 15) $\chi = 28$

16) $\chi = 6$

Page 90, **Solving Similar Triangles**

1) A = 2 500 mm 2) B = 3 mm

3) C = 6 000 mm 4) D = 189.6 mm

5) E = 652 mm 6) F = 625 mm

7) G = 400 mm
H = 600 mm

8) 15 ÷ 6 = 2.5 mm
I = 2 mm
J = 4 mm
K = 6 mm
L = 8 mm
M = 10 mm

9) N = 2.3 mm 10) P = 546.88 mm

11) Q = 1 485 mm 12) R = 750 mm

Page 97, **Solving an Indirect Proportion**

1) $\frac{5}{9} = \frac{\chi \text{ days}}{80}$

$\frac{400}{9} = \chi$

$44.4 = \chi$

2) $\frac{8}{11} = \frac{\chi \text{ days}}{26}$

$\frac{208}{11} = \chi$

$18.9 = \chi$

3) $\frac{220 \text{ lbs applied force}}{\chi} = \frac{6 \text{ in lever arm}}{58 \text{ in lever arm}}$

$\frac{12\ 760}{6} = \chi$

$2\ 126.7 = \chi$

4) $\frac{180 \text{ lbs applied force}}{\chi} = \frac{4.5 \text{ in}}{55.5 \text{ in}}$

$\frac{9\ 990}{4.5} = \chi$

$2\ 200 = \chi$

Page 99, **Using Rates to Compare**

1) Journeyperson $\frac{2}{3} = 0.667$ Apprentice $\frac{2}{6.75} = 0.296$

$0.667 + 0.296 = 0.963$ hours

$$\frac{\chi}{2 \text{ doors}} = \frac{60 \text{ minutes}}{0.963 \text{ hours}}$$

$\chi \times 0.963 = 60 \times 2$

$\chi = \frac{120}{0.963}$

$\chi = 124.6106$ minutes

$124.6106 \div 60 = 2.0768$ hours

$0.0768 \times 60 = 4.6$ minutes

It will take the journeyperson and the apprentice 2 hours and 5 minutes to install 2 doors.

Page 100, **Practice**

1) $\frac{7}{2} = \frac{12 \text{ m}^3}{\chi}$

$\chi = \frac{24}{7} = 3.4 \text{ m}^3$

2)a. $\frac{1}{0.75} = \frac{\chi}{6}$

$\frac{6}{0.75} = \chi$

$8' = \chi$

b. $\frac{1}{0.75} = \frac{\chi}{8.5 \text{ feet}}$

$\frac{8.5}{0.75} = \chi$

$11.3 \text{ feet} = \chi$

c. $\frac{1}{0.75} = \frac{2\ 500\ \text{mm}}{\chi}$

$2\ 500 \times 0.75 = \chi$

$1\ 875\ \text{mm} = \chi$

d. $\frac{1}{0.75} = \frac{3\ 200\ \text{mm}}{\chi}$

$3\ 200 \times 0.75 = \chi$

$2\ 400\ \text{mm} = \chi$

3) $\frac{3.6\ \text{kg}}{90\ \text{m}^2} = \frac{\chi}{2\ 500\ \text{m}^2}$

$3.6 \times 2\ 500\ \text{m}^2 = \chi \times 90\ \text{m}^2$

$\frac{9\ 000}{90} = \chi$

$100\ \text{kg} = \chi$

4)

	A	B	C	D
a.	15 mm	20 mm	54 mm	72 mm
b.	180 mm	240 mm	480 mm	640 mm
c.	35 m	45 m	105 mm	135 m
d.	22 m	38 m	48 m	82.9 m

5) cement

$\frac{1}{10} = \frac{\chi}{86\ \text{m}^3}$

$\frac{86}{10} = \chi$

$8.6\ \text{m}^3$ cement

sand

$\frac{3}{10} = \frac{\chi}{86\ \text{m}^3}$

$\frac{258}{10} = \chi$

$25.8\ \text{m}^3$ sand

gravel

$\frac{6}{10} = \frac{\chi}{86\ \text{m}^3}$

$\frac{516}{10} = \chi$

$51.6\ \text{m}^3$ gravel

6) $\frac{6 \text{ carpenters}}{4 \text{ carpenters}} = \frac{\chi \text{ hours}}{5 \text{ hours}}$

$6 \times 5 = \chi \times 4$

$\frac{30}{4} = \chi$

$7.5 \text{ hours} = \chi$

7) $\frac{8 \text{ carpenters}}{\chi \text{ carpenters}} = \frac{4 \text{ days}}{5 \text{ days}}$

$8 \times 5 = 4 \times \chi$

$\frac{40}{4} = \chi$

$10 \text{ carpenters} = \chi$

8) $\frac{3}{5} = \frac{\chi}{8.75}$ $\qquad$ $\frac{4}{5} = \frac{\chi}{8.75}$

$3 \times 8.75 = \chi \times 5$ $\qquad$ $4 \times 8.75 = \chi \times 5$

$\frac{26.25}{5} = \chi$ $\qquad$ $\frac{35}{5} = \chi$

$5.25 \text{ cm} = \chi$ $\qquad$ $7 \text{ cm} = \chi$